

Geraldton Senior High School

**ATMAM Math Methods**

Test 1: Chapters 1 to 5

**20 minutes Section 1 - Calculator Free 20 marks**

**SOLUTIONS**

1. **[6 marks]**

A body moves under rectilinear motion such that for time seconds, , the body’s displacement, metres, from the origin O is given by

* 1. Determine its acceleration at . (2 mark)

|  |  |
| --- | --- |
| Solution | Specific Behaviours |
|  | * Uses first derivative to determine velocity. * Uses second derivative to determine acceleration and evaluates at . |

* 1. Use calculus techniques and the second derivative to determine and justify when the displacement is at a maximum.  
      (4 marks)

|  |  |
| --- | --- |
| Solution | Specific Behaviours |
|  | * Recognizes that max displacement occurs when and determines . * Solves . * Determines and uses Second Derivative Test to identify max. at . * Substitutes into to determine maximum displacement. |

1. **[5 marks]**

For the following questions, **do not simplify your answers.**

* 1. Determine (3 marks)

|  |  |
| --- | --- |
| Solution | Specific Behaviours |
|  | * Determines * Determines * Uses Quotient Rule correctly and divides by |

* 1. Determine given that (2 marks)

|  |  |
| --- | --- |
| Solution | Specific Behaviours |
|  | * Determines * Determines using the Product Rule correctly. |

1. **[5 marks]**

Determine

* 1. (2 marks)

|  |  |
| --- | --- |
| Solution | Specific Behaviours |
|  | * Writes the integrand as two separate fractions for purposes of integrating. * Uses the additivity of integration to correctly integrate. |

* 1. (3 mark)

|  |  |
| --- | --- |
| Solution | Specific Behaviours |
|  | * Uses understanding of chain rule to obtain correct integral. * Substitutes the given bounds correctly as per FTC part 2. * Correctly evaluates the definite integral. |

1. **[4 marks]**

Consider the functions given by and .

Determine:

* 1. (1 mark)

|  |  |
| --- | --- |
| Solution | Specific Behaviours |
|  | * Determines |

* 1. (1 mark)

|  |  |
| --- | --- |
| Solution | Specific Behaviours |
|  | * Uses the chain rule (short form) to determine |

And hence,

* 1. Use parts a) and b) to determine in its simplest form. (2 marks)

|  |  |
| --- | --- |
| Solution | Specific Behaviours |
|  | * Uses chain rule in long form to express * Correctly simplifies the product. |

NAME: TEACHER:

Geraldton Senior High School

**ATMAM Math Methods**

Test 1: Chapters 1 to 5

**35 minutes Section 2 - Calculator Assumed 35 marks**

1. **[4 marks]**

A sphere of radius 30 cm is sitting on the base of a container of water. The water level is rising at a rate of 2 cm per second, so that the sphere is gradually becoming submerged.

Let h be the height of the water as illustrated below left. The submerged portion of the sphere is as shown below right.

Diagram, venn diagram

Description automatically generated A picture containing diagram

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The volume of the submerged portion, where is the radius of the sphere is given by

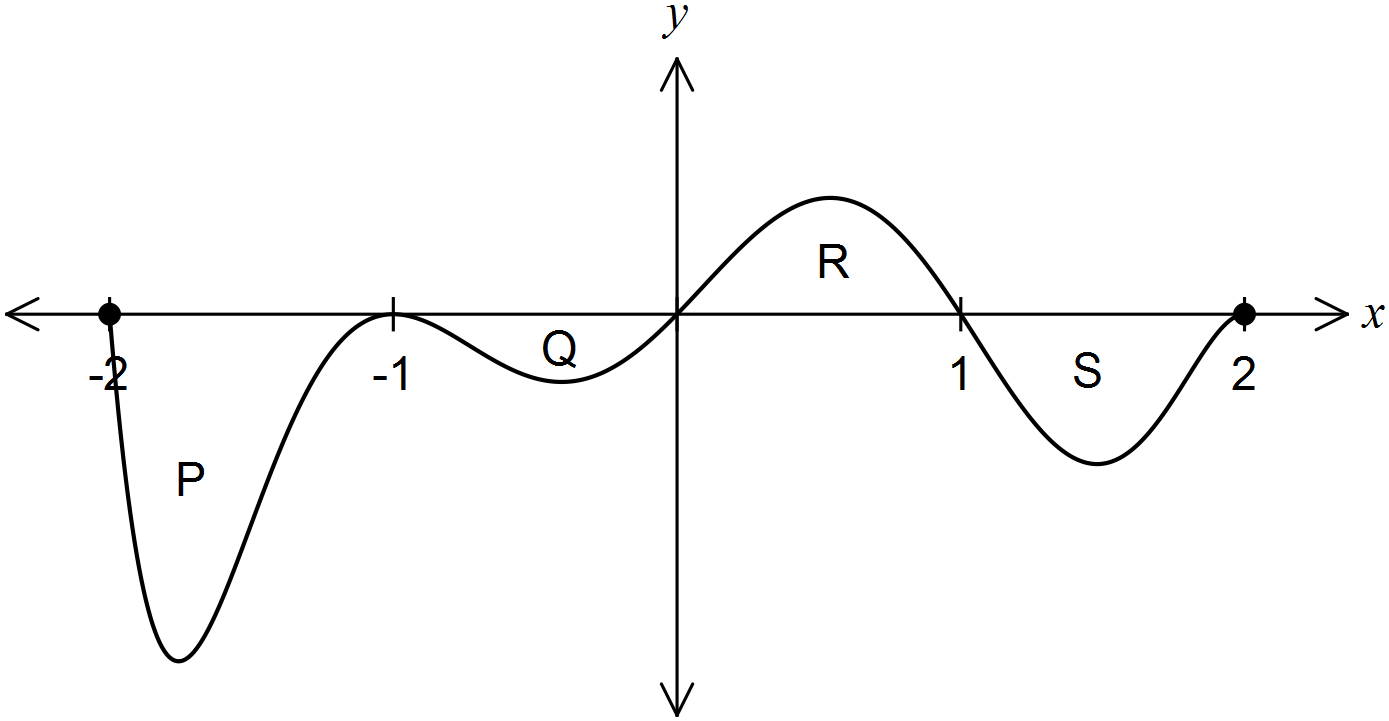
Using the small change formula: or equivalently

Calculate to 1d.p. the approximate increase in submerged volume for a increase in height when

|  |  |
| --- | --- |
| Solution | Specific Behaviours |
|  | * Determines (with or without initially substituting * States * Correctly applies the incremental change formula. * Correctly evaluates substituted values. |

1. **[6 marks]**

The graph of the function is shown below over the domain .



The areas of regions P, Q, R and S enclosed by the curve and the are 5, 1, 2 and 3 square units respectively.

* 1. Determine the area enclosed by the curve and the for . (1 mark)

|  |  |
| --- | --- |
| Solution | Specific Behaviours |
|  | * Correctly sums the indicated areas. |

* 1. Determine the value of

1. (1 mark)

|  |  |
| --- | --- |
| Solution | Specific Behaviours |
|  | * Correctly sums the signed areas to obtain the definite integral. |

1. (2 marks)

|  |  |
| --- | --- |
| Solution | Specific Behaviours |
|  | * Recognises a horizontal translation at play * Sums the signed areas to obtain the Definite Integral |

1. (2 marks)

|  |  |
| --- | --- |
| Solution | Specific Behaviours |
|  | * Uses additivity of integration to separate the integral. * Uses the result from part i) and the definite integral to obtain the final result. |

1. **[7 marks]**

A cylinder of radius metres and height metres is to be constructed inside a cone of height and base radius as shown.



* 1. Show clearly why . (1 mark)

*6*

*6–h*

*r*

*3*

*h*

|  |  |
| --- | --- |
| Solution | Specific Behaviours |
|  | * Uses similar triangles to obtain the ratio and isolates |

* 1. Show clearly why the volume of the cylinder is given as: (2 marks)

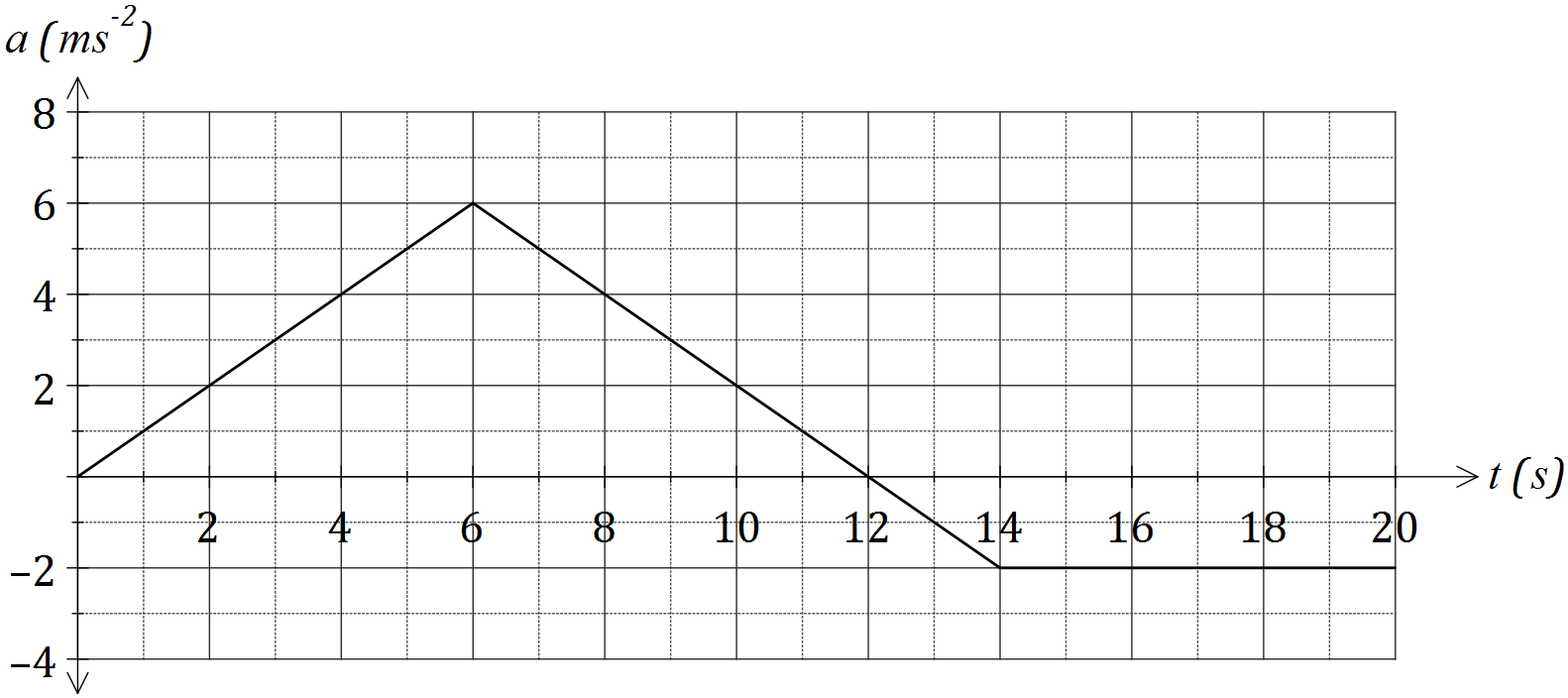
|  |  |
| --- | --- |
| Solution | Specific Behaviours |
|  | * Substitutes into * Expands and simplifies to obtain correct formula |

* 1. Use calculus to determine the dimensions (i.e. height & radius) of such a cylinder which gives maximum volume. (4 marks)

|  |  |
| --- | --- |
| Solution | Specific Behaviours |
|  | * Recognizes that volume is maximised at and determines * Solves * Uses the Second Derivative to justify that a maximum is obtained at * Substitutes into to determine correct dimensions. |

1. **[8 marks]**

A particle, initially stationary and at the origin, moves subject to an acceleration, ms-2, as shown in the graph below for seconds.



(a) Determine the velocity of the object when

1. . (1 mark)

|  |  |
| --- | --- |
| Solution | Specific Behaviours |
|  | * Identifies using initial conditions and area to determine velocity |

1. . (2 marks)

|  |  |
| --- | --- |
| Solution | Specific Behaviours |
|  | * Identifies using initial conditions * Sums signed areas to determine velocity |

(b) At what time is the velocity of the body a maximum, and what is the maximum velocity?   
 (2 marks)

|  |  |
| --- | --- |
| Solution | Specific Behaviours |
|  | * Identifies maximum velocity at * Determines area to calculate maximum velocity |

(c) Determine the distance of the particle from the origin after seconds. (3 marks)

|  |  |
| --- | --- |
| Solution | Specific Behaviours |
|  | * Integrates using initial conditions to determine * Integrates using initial conditions to determine * Evaluates at to determine displacement. |

1. **[5 marks]**

Leveraging your calculator use the calculus tools of thefirst and second derivative to determine the location and nature of all the stationary points of

A picture containing background pattern

Description automatically generatedGraphical user interface, application

Description automatically generated with medium confidence

**✓**

**✓**

**✓**

**✓**

**✓**

1. **[5 marks]**

Shown are the graphs of



*b*

*a*

* 1. Write an appropriate calculation using three integrals that can be used to determine the area bound by the two curves and . Do not evaluate. (3 marks)

|  |
| --- |
| Solution |
|  |
| Specific Behaviours |
| * Solves to obtain intersections not apparent in graph. * Writes three integral with appropriate bounds * Places absolute values around each integral |

* 1. Calculate the area bound between the curves and by writing and evaluating a single integral over the bounds, . (2 marks)

|  |  |
| --- | --- |
| Solution | Specific Behaviours |
|  | * Writes a single integral with correct bounds and absolute values around the integrand * Calculates the bounded area. |